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Merging States and Synchronization Problem[1]

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Abstract. In this paper we introduce the notion of merging states and merging systems and we use it for the classification of finite deterministic automata without initial and final states. We investigate the dependencies between the structure of an automaton described by merging systems and maximal lengths of minimal synchronizing words for automata which structures belong to the given class of merging systems. Numerical results for certain classes of automata are presented. We also give some properties of merging systems themselves. The work is motivated by the famous, unsolved Černý Conjecture. The aim of this paper is to propose the use of merging systems in the research on the Conjecture.

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1. Introduction and motivation

Let $\mathcal{A} = (Q, A, \delta)$ be a finite, deterministic and complete automaton without initial or final states. If there is a word w such that for all states

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$p, q \in Q$ we have $p.w = q.w$ then we say that w synchronizes $\mathcal{A}$ and call such automaton a synchronizing one. If there is no shorter word we call w the minimal synchronizing word for $\mathcal{A}$. We denote by $\mathcal{M}(\mathcal{A})$ the length of the minimal synchronizing word for $\mathcal{A}$.

Černý Conjecture states that for n-state synchronizing automaton $\mathcal{A}$ we have $\mathcal{M}(\mathcal{A}) \leq (n-1)^2$. The Conjecture was stated in 1964 and is still open. Černý proved that for each n there exists the n-state automaton which possesses the minimal synchronizing word of length exactly $(n-1)^2$, therefore obtaining the upper bound from the Conjecture. Such automata are so-called Černý automata.

A synchronization problem – that is finding synchronizing words – seems to be only a nice combinatorial puzzle, but in fact it has many important applications in the industry (particulary in so-called part orienters [12]), biocomputing (reset problem [3]), network theory, etc. Therefore the problem is of general interest. We would like to know if the $(n-1)^2$ bound is optimal (there is a gap between this $O(n^2)$ conjectured bound and the best bound known so far, $O(n^3)$) and what method would be the best for finding the shortest possible synchronizing sequences. There are some polynomial algorithms but the problem with finding the *minimal* synchronizing word is NP-complete, so probably there arenot any polynomial algorithms which can work in the optimal way in all cases.

The reader is referred to [1, 3, 10, 12] for more details on the role of the synchronization problem and to [7, 16] for polynomial algorithms and the proof of NP-completeness.

The Černý Conjecture turned to be true for some special cases (see for example [1, 2, 5, 6, 7, 9, 15, 16]) but in general case it is still open. The best known upper bound for $\mathcal{M}(\mathcal{A})$, where $\mathcal{A}$ has n states, is $\frac{n^3-n}{6}$ [11].

In this paper, motivated by the Černy Conjecture, we introduce the notion of merging states and merging systems in finite automata and use them to investigate the dependencies between the structure of an automaton described by a merging system and maximal lengths of minimal synchronizing words for automata which structures belong to the given class of merging systems. Numerical results for certain classes of automata are presented. We also give some properties of merging systems themselves. The aim of this paper is to propose the use of merging systems in the research on the Conjecture.

2. Merging states

We will consider finite automata defined as triples $\mathcal{A} = (Q, A, \delta)$, where Q is a finite set of states, A is a finite alphabet and $\delta : Q \times A \to Q$ is a function transforming states. It can be extended on a free monoid A^* and set of subsets of Q:

$$\delta(P, aw) = \bigcup_{p \in P} \delta(\delta(p, a), w) \;\; a \in A, \;\; w \in A^*.$$

By $\delta^{-1}(q, a)$ we understand the set of states which come to q under the letter a: $\delta^{-1}(q, a) = \{p \in Q : \;\delta(p, a) = q\}$. This function can be also extended:

$$\delta^{-1}(P, w) = \{q : \;\delta(q, w) \in P\}, \;\; P \subseteq Q, \;\; w \in A^*.$$

Let $\mathcal{A}$ be an automaton. We say that w is a *synchronizing word* for $\mathcal{A}$ if $|\delta(Q, w)| = 1$. An automaton which admits a synchronizing word is called a *synchronizing automaton.* If w is a synchronizing word and there is no shorter one, w is called the *minimal synchronizing word.*

Definition 1. *State $q \in Q$ is a merging state of degree k for $a \in A$ if $|\delta^{-1}(q, a)| = k$.*

In other words it means that there exist k different states $p_1, ..., p_k \in Q$ such that $\delta(p_i, a) = q$ and these are the only states, which comes to q under the letter a. The set of states $\{p_1, ..., p_k\}$ will be called the *merging system* (for a, of q and of degree k) and q will be called the merging state for $\{p_1, ..., p_k\}$.

Note that q can, although it hasnot to be, one of the p_i's. We will differ these two situations:

Definition 2. *Let $P = \{p_1, ..., p_k\}$ be a merging system for a merging state q. If $\exists i : q = p_i$, then q will be called an internal merging state for P and the set P – an internal merging system.*

An *external merging system* is a system which is not internal. In this situation we have $q \neq p_i \forall i = 1, ..., k$ and we say that q is an *external merging state* for P.

In Fig. 1 one can see the examples of two merging systems described above.

Let us assume that $P = \{p_1, ..., p_k\}$ is a merging system for $a \in A$ or $q \in Q$ and let q be an external merging state for P. Then q belongs to another merging system $R = \{r_1, ..., r_t\}$ ($P \cap R = \emptyset$) for the same letter a but cannot be the merging state for R. If s is a merging state for R then $q \neq s$

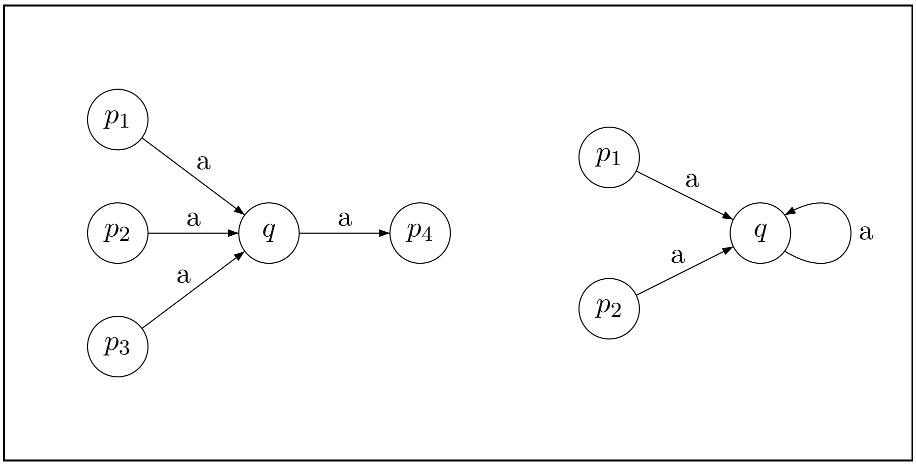

Fig. 1. External and internal merging systems of degree 3

only if $q \in R$. In contradiction, if $q = s$ then P and R would have to be one merging system $P = R = \{p_1, ..., p_k, r_1, ..., r_t\}$ with a merging state $q = s$. It comes directly from the definition of a merging state and determinism of automaton: each state, for a given letter, can be a merging state only for one merging system (see Proposition 1).

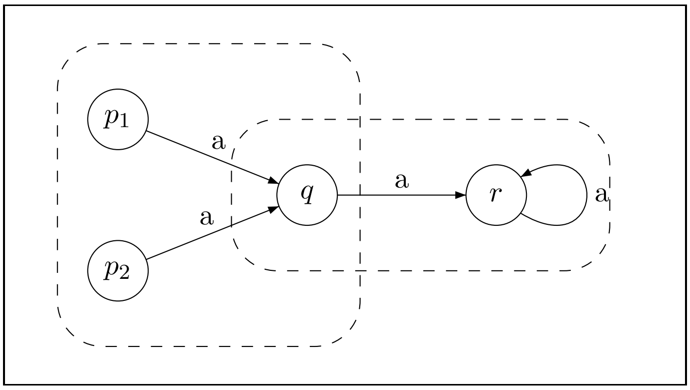

Fig. 2. q as a merging state and in the same time as a part of other merging system

Fig. 2 illustrates this situation: q is a merging state for $\{p_1, p_2\}$ and belongs to another merging system $\{q, r\}$.

In the following we will assume that each time merging systems will be considered for a fixed letter $a \in A$. We also assume that all automata are deterministic and complete (it means that δ is a total function).

A merging state of degree greater than 1 will be called a proper one.

Let $\mathcal{A} = (Q, A, \delta)$, where $|Q| = n$, $|A| = m$. Let us introduce the notation for describing all merging systems in $\mathcal{A}$:

DEFINITION 3. *Let $\mathcal{A}$ for a_i, $i = 1, \ldots, m$ has $\lambda_t^{a_i}$ external merging systems of degree t, $\mu_t^{a_i}$ internal merging systems of degree t and maximal degree of merging systems doesnot exceed k_i. Then such automaton will be described in the following way:*

$$\mathcal{A} \sim [1^{\lambda_1^{a_1}} 1_*^{\mu_1^{a_1}} \ldots k_1^{\lambda_{k_1}^{a_1}} k_{1*}^{\mu_{k_1}^{a_1}}]_{a_1} \ldots [1^{\lambda_1^{a_m}} 1_*^{\mu_1^{a_m}} \ldots k_m^{\lambda_{k_m}^{a_m}} k_{m*}^{\mu_{k_m}^{a_m}}]_{a_m}.$$

The above notation will be called a merging type of $\mathcal{A}$.

EXAMPLE 1. *Let's take $n = 4$, $k = 3$, $n - k = 1$ and fix the partitions: $k = 3 = 1 + 2$, $n - k = 1$. The corresponding merging type is $[11_*2_*]$ and it is represented in an automaton in Fig. 3.*

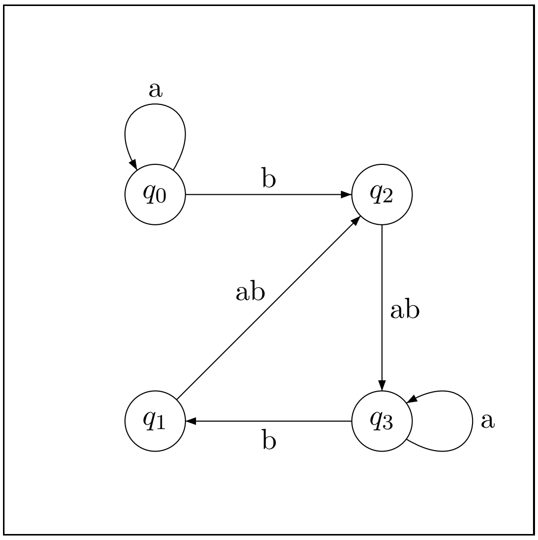

Fig. 3. An automaton with merging type $[11_*2_*][1^2 2]$

In the paper we will sometimes use the following convention: if there arenot any proper merging systems for a given letter then this part of the notation will be omitted. Also (especially in the section with numerical results) the notation of systems of degree 1 will be omitted. This convention will be used if there is no need for distinguishing external and internal merging systems of degree 1.

The following proposition and theorem are true. The proposition claims that each state is involved in exactly one merging system, the theorem gives the generating function for the formula counting the number of syntactically different merging states for a single letter (that means, the number of different notations like the one introduced in Definition 3).

PROPOSITION 1. *If the merging type for* $\mathcal{A} = (Q, A, \delta)$, $|Q| = n$ *is* $[1^{\lambda_1} 1_*^{\mu_1} ... k^{\lambda_k} k_*^{\mu_k}]$ *then*

$$\sum_{i=1}^{k} i * (\lambda_i + \mu_i) = |Q| = n.$$

□

THEOREM 1. *The number* L *of different merging types for a letter* a *in automaton* $\mathcal{A} = (Q, A, \delta)$, $|Q| = n$ *is*

$$L = \sum_{k=0}^{n} ([x^k] \prod_{i=1}^{n} (1 - x^i)^{-1} * [x^{n-k}] \prod_{j=1}^{n} (1 - x^j)^{-1}) - 1,$$

where $[x^n]$ *denotes the coefficient at* x^n.

PROOF. Let's divide the set of states into two classes K_1 and K_2: states, which belong to internal merging systems will be in K_1, states which belong to external ones will be in K_2. Obviously $K_1 + K_2 = |Q| = n$. Let $|K_1| = k$. This k states can be divided into t groups of $k_1, k_2, ..., k_t$ elements, $k = k_1 + k_2 + ... + k_t$. The i-th group (with k_i elements) represents the internal merging system of degree k_i. The same situation is with $n - k$ remaining states – but here groups represent external merging systems ($n - k = l_1 + l_2 + ... + l_s$). One can see that each such partition can stand for the partition of integers k and $n - k$. Note that each pair of partitions for k and $n - k$ (here: $[l_1 l_2 ... l_s k_{1*} k_{2*} ... k_{t*}]$) has some representation in merging systems, although an automaton with such structure is not a synchronizing one in general. Furthermore, for a given partition of k and $n - k$ there can be more than one automaton which structure is induced by this partition (up to isomorphism). Because k was chosen arbitrarily, we consider all possible pairs of partitions of k and $n - k$ and we sum the corresponding coefficients in generating functions which represent above partitions. We must subtract 1 from the result because one partition isnot possible (for $k = 0$ and partition n into one part there is no external merging system). □

One can bring out the formula which describes the dependence between the merging system and the transition function for a given automaton (we give here the version for merging only for one letter, for the sake of simplicity).

PROPOSITION 2. *Let $\mathcal{A} = (Q, A, \delta)$ be of type $[1^{\lambda_1} 1_*^{\mu_1} ... k^{\lambda_k} k_*^{\mu_k}]_a$. Then*

$$|Q| - |\delta(Q, a)| = \sum_{i=2}^{k} (i-1)(\lambda_i + \mu_i).$$

PROOF. Let $Q = P_1 \cup P_2 \cup ... \cup P_k \cup R_1 \cup R_2 \cup ... \cup R_k$, where P_j (resp. R_j) represents the union of all internal (resp. external) merging systems of degree j. From Proposition 1 we know that the cross section of any two of these sets is empty. Furthermore, if $P_j = P_j^1 + ... + P_j^{\lambda_j}$, where P_j^k is k-th merging system of degree j, the cross section of any two sets P_j^x and P_j^y is empty. It is obvious that for given j we have $|P_j^1| = |P_j^2| = ... = |P_j^{\lambda_j}| = j$. For each P_i^j we have $|\delta(P_i^j, a)| = 1$ and for each P_i we have $|\delta(P_i, a)| = \lambda_i$. Therefore $|P_i^j| - |\delta(P_i^j, a)| = |P_i^j| - 1$. The same facts are true for each R_i. We have:

$$\begin{aligned}
|Q| - |\delta(Q,a)| &= |\bigcup_{i=1}^{k}(P_i \cup R_i)| - |\delta(\bigcup_{i=1}^{k}(P_i \cup R_i), a)| \\
&= \sum_{i=1}^{k} |P_i \cup R_i| - \sum_{i=1}^{k} |\delta((P_i \cup R_i), a)| \\
&= \sum_{i=1}^{k} ((|P_i| - |\delta(P_i, a)|) + (|R_i| - |\delta(R_i, a)|)) \\
&= \sum_{i=1}^{k} ((|P_i| - \lambda_i) + (|R_i| - \mu_i)) \\
&= \sum_{i=1}^{k} ((\lambda_i * |P_i^1| - \lambda_i) + (\mu_i * |R_i^1| - \mu_i)) \\
&= \sum_{i=1}^{k} (\lambda_i * (i-1) + \mu_i * (i-1)) \\
&= \sum_{i=1}^{k} (i-1)(\lambda_i + \mu_i) \\
&= \sum_{i=2}^{k} (i-1)(\lambda_i + \mu_i).
\end{aligned}$$

$\square$

3. Černý Conjecture

Černý Conjecture was stated in 1964 [4] and claims that the length of the minimal synchronizing word for the n-state synchronizing automaton is not greater than $(n-1)^2$. It is known to be true for special cases (see [1, 5, 6, 7, 9]) but we do not know if it is true in general case. The best known general upper bound is $\frac{n^3-n}{6} - 1$ [11, 13]. For a good survey on the problem see the paper of Salomaa [14].

Let $\mathcal{M}(\mathcal{A})$ be the length of the minimal synchronizing word for $\mathcal{A}$ and let $\mathcal{M}([1^{\lambda_1}...k_*^{\mu_k}])$ be the longest word among minimal synchronizing words for automata of a given type.

It is known fact that for each $n \geq 2$ there exists the n-state automaton for which the conjectured upper bound $(n-1)^2$ is obtained. These are so-called 'Černý automata'. The transition function for the n-state Černý automaton with $Q = \{0, 1, ..., n-1\}$ and $A = \{a_0, a_1\}$ is defined as follows:

$$\delta(q, a) = \begin{cases} q+1 \pmod{n} & \text{for } a = a_0 \\ q & \text{for } a = a_1 \wedge q \neq n-1 \\ 0 & \text{for } a = a_1 \wedge q = n-1. \end{cases}$$

It turned out that for some n there exist automata which are non-isomorphic with Černý's ones but also reach conjectured bound ($\mathcal{M}(\mathcal{A}) = (n-1)^2$). It is easy to give an example for $n = 2, 3$. Černý gave the example for $n = 4$. We found it for $n = 6$ – it is the same automaton which was found by J. Kari. He gave this example as the counterexample of Pin's extended Conjecture [8]. For $n = 5, 7, 8$ there are no such automata for a 2-letter alphabet. Up to this time it is unknown if each automaton of type $[2_*]$ fulfills Černý Conjecture, although it is known that for each n there exists such automaton of this type for which the conjecture is true and the upper bound is obtained (Černý automaton).

4. Numerical results. Merging states and synchronization

In the table we present the maximal lengths of minimal synchronizing words for automata of certain classes. These classes are defined by merging types – $[k^1]$ or $[k_*^1]$ for $k = 2, ..., n-1$ for given n (it means that k states produce one merging system and other $n-k$ states are 'single' merging systems of degree 1). Results were found by computer for $n = 2, 3, ..., 9$.

Merg. type / n	2	3	4	5	6	7	8	9
2_*	1	4	9	16	25	36	49	64
2		4	9	15	25	32	44	58
3_*		1	4	9	16	25	36	49
3			4	9	17	25	33	44
4_*			1	4	9	16	25	36
4				4	10	18	28	37
5_*				1	4	9	16	25
5					4	11	21	31
6_*					1	4	9	16
6						4	12	22
7_*						1	4	9
7							4	13
8_*							1	4
8								4
9_*								1

Fig. 4. Numerical results for $n = 2, 3, ..., 9$

We will prove now that the 'boundary' values from the table are in fact 1, 4 and 4.

Theorem 2. $\mathcal{M}([n_*]) = 1$.

Theorem 3. $\mathcal{M}([n-1]) = 4$.

Theorem 4. $\mathcal{M}([(n-1)_*]) = 4$.

Proof of Theorem 2. It's trivial – for each automaton of type $[n_*]_a$ $|\delta(Q, a)| = 1$ and $|a| = 1$. □

Proof of Theorems 3 and 4. Let $a \in A$ be the letter which synchronizes $n-1$ states. Let awa be the shortest word which synchronizes $\mathcal{A}$, $w = b_1...b_m$. Suppose $m \geq 3$. Then the kernels of $\delta(\_, b_2...b_m a)$, $\delta(\_, b_3...b_m a)$ and $\delta(\_, a)$ are three different partitions of Q into two parts. If two of them, say $\delta(\_, b_2...b_m a)$ and $\delta(\_, b_3...b_m a)$ coincide, then the word $ab_1b_3...b_m a$ would synchronize $\mathcal{A}$, a contradiction. On the other hand, the 2-element image of Q under $\delta(\_, a)$ should be a cross-section of each of the three partitions – indeed, if the image is not a cross-section of the kernel of $\delta(\_, b_2...b_m a)$, say, then the word

$ab_2...b_m a$ synchronizes $\mathcal{A}$, a contradiction. But one of the two parts of each of the three partitions must be a singleton and since the partitions are different, their singleton parts are different as well. So, no triple of such partitions can share a common cross-section. Thus $m \leq 2$, so the length of the shortest synchronizing word cannot be greater than 4. □

For merging type $[(n-2)]$ it is easy to show that $\mathcal{M}(A) \geq n+4$ for $n > 4$: the transition function for the automaton with this property is defined as follows: $\delta = (211...1n)(1345...(n-1)n2)$: states $2, 3, ..., n-1$ merges into state 1 under a, b induces a $(n-1)$-cycle on states $2, 3, ..., n-1, n$ and 1-cycle on state 1.

One can ask if there are some relations between the structure of an automaton and the length of its minimal synchronizing word. Particulary we would like to know if there are relations saying how the length of the minimal synchronizing word changes if we change the type of an automaton. In general, there are no such relations:

Let $\mathcal{A} = (Q, A, \delta)$ be a synchronizing automaton and $\mathcal{A}' = (Q, A, \delta')$ – an automaton given by changing one value in δ from $\mathcal{A}$:

$$\delta'(q, a) = \begin{cases} \delta(q, a) & \text{for } q \neq q_0 \vee a \neq a_0 \\ q' & \text{for } q = q_0 \wedge a = a_0, \end{cases}$$

where $q' \neq \delta(q_0, a_0)$.

Now we give 3 examples showing that in general case (that means, if we change the transition function arbitrarily) there is no dependence between changing one value in δ and the length of the minimal synchronizing word for this new automaton. The notation for δ as $(a_1...a_n)(b_1...b_n)$ means: $\delta(q, a) = a_q$. For example, if $\delta = (231)(112)$, then $\delta(q_3, b) = 2$, $\delta(q_2, a) = 3$ etc. In all 3 examples the merging type changes from $[1^2 3_*]_b$ into $[122_*]_b$.

Example 2. *Let* $\delta = (31524)(33354)$, $\delta' = (31524)(34354)$. *Then* $\mathcal{M}(\mathcal{A}) = 5 < 7 = \mathcal{M}(\mathcal{A}')$. *Minimal synchronizing words for* $\mathcal{A}$ *and* $\mathcal{A}'$ *are baaab and baaabab, respectively.*

Example 3. *Let* $\delta = (53412)(33354)$, $\delta' = (53412)(35354)$. *Then* $\mathcal{M}(\mathcal{A}) = 6 > 5 = \mathcal{M}(\mathcal{A}')$. *Minimal synchronizing words for* $\mathcal{A}$ *and* $\mathcal{A}'$ *are babaab and babab.*

Example 4. *Let* $\delta = (52143)(33354)$, $\delta' = (52143)(34354)$. *Then* $\mathcal{M}(\mathcal{A}) = 5 = 5 = \mathcal{M}(\mathcal{A}')$. *The minimal synchronizing word for* $\mathcal{A}$ *and* $\mathcal{A}'$ *is babab.*

From the other side we see that there is some regularity in the table and it seems that there are relations such as mentioned above, if we consider changing the number of states: the greater number of states is, the longest is the minimal synchronizing word (in the given class of merging type). The next step is to see if there are relations between the number of precisely merging systems and the lengths of minimal synchronizing words.

In our computations we were interested only in automata with one merging system, because probably this is the sufficient condition which should be fulfilled by n-state automata in order to find the minimal synchronizing word of maximal length for a given n. Namely, we strongly believe in the following conjectures.

CONJECTURE 1. *If $\mathcal{A}$ is an n-state synchronizing automaton with k merging systems $S_1, S_2, ..., S_k$ $(k > 1)$, then there exists a synchronizing n-state automaton $\mathcal{B}$ with $k-1$ merging systems $S_1, S_2, ..., S_{k-1}$ such that*

$$\mathcal{M}(\mathcal{A}) \leq \mathcal{M}(\mathcal{B}).$$

CONJECTURE 2. *If $\mathcal{A}$ is an n-state synchronizing automaton with merging system of degree k $(k > 2)$, then there exists a synchronizing n-state automaton $\mathcal{B}$ with merging system of degree $k-1$ such that*

$$\mathcal{M}(\mathcal{A}) \leq \mathcal{M}(\mathcal{B}).$$

It intuitively seems that the first conjecture should be true. Computations support the second conjecture: computer, for 4, 5 and 6-state automata with one merging system P of degree greater than 2 found that for all of them, if q^* is the merging state and $R = \{q \in Q : \delta^{-1}(q, a) = \emptyset\}$, then there always exist state p from P and r from R such that if we change the value $\delta(p, a)$ from q^* into r then the length of the minimal synchronizing word for the new automaton is greater than the length of such word for the initial automaton.

Because we ask if Černý Conjecture is true, assuming the conjectures 1 and 2 are true it would be enough to prove Černý Conjecture only for automata with one merging system of degree 2, so the problem would reduce into one case:

PROPOSITION 3. *If Conjectures 1 and 2 are true, then Černý Conjecture holds iff $m(\mathcal{A}) \leq (n-1)^2$ for each $\mathcal{A}$ of merging type* [2] *and* $[2_*]$.

PROOF. Let $\mathcal{A}_k$ be an automaton with k merging systems. Then, according to Conjecture 1, we can build the sequence of automata $\mathcal{A}_k$, $\mathcal{A}_{k-1}$, ..., $\mathcal{A}_1$ such that $\mathcal{A}_i$ has i merging systems and $\mathcal{M}(\mathcal{A}_i) \geq \mathcal{M}(\mathcal{A}_j)$ for $i < j$.

Then, applying the Conjecture 2 to the automaton $\mathcal{A}_1 = \mathcal{A}_1^t$ with one merging system of degree t we can again build the sequence $\mathcal{A}_1^t$, ..., $\mathcal{A}_1^2$ such that $\mathcal{A}_1^i$ has one merging system of degree i and $\mathcal{M}(\mathcal{A}_1^i) \geq \mathcal{M}(\mathcal{A}_1^j)$ for $i < j$. For automaton $\mathcal{A}_k$ we found automaton $\mathcal{A}_1^2$ such that $\mathcal{M}(\mathcal{A}_1^2) \geq \mathcal{M}(\mathcal{A}_k)$. So the problem reduces into automata with only one merging system of degree 2. $\square$

We would also like to investigate the dependencies between these values for different merging types. This is why we did the computations for automata with one merging system but of different degrees.

5. Summary

We presented the classification of finite automata according to the merging type, described properties of merging systems and showed the results of computations with aim to find the maximal lengths of minimal synchronizing words for automata of certain types. We proved three theorems on the (constant for each n) lengths of minimal synchronizing words for automata of type $[n]$, $[n-1]$, $[(n-1)_*]$ and gave some lower bound for type $[n-2]$ which seems to be also the upper bound. We stated some open problems and showed that if Conjecture 1 and 2 are true, Černý Conjecture will reduce to one case – the family of automata of type $[2]$ and $[2_*]$.

We think that the notion of merging systems, if developed more, can be a good tool in the research on a synchronization problem. We are interested in the following problem: what are the relations between values in the table from Section 3 and how does the length of the minimal synchronizing word behave if we change the stucture of the automaton by adding or removing one state or changing the merging type by redirecting one arrow in the automaton? Answers to these questions can lead to some new facts about synchronization, give some wide class of automata fulfilling the Conjecture, reducing the Conjecture to the simpler problem or even prove or disprove the Conjecture.

The results from this paper were presented at the ICALP'04, Workshop on Synchronizing Automata in Turku, Finland.

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